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THE NONLINEAR STATIC ANALYSIS FOR VISCOELASTIC ROD SUBJECTED TO CONSTANT AXIAL LOAD

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Abstract: This paper presents the results of the nonlinear static analysis for a beam subjected to constant axial load. The beam is fixed on axial direction at one end and on the othe r end is applied a constant axial load. The beam is considered to be from linear viscoelastic material. There are used in finite element modeling SOLID type of element for modeling the rod. There are presented the strain and stress distribution in time after performing nonlinear analysis because of the nonlinearity of the material.

1. THEORETICAL OVERVIEW ABOUT VISCOELASTIC BEHAVIOUR

The tendency of replacing of metallic materials – steel – with rheological materials with reduced inertial forces and couples because of their lower specific mass goes to developing of the research in rheological domain. Generalizing the Hooke elasticity theory and the viscous fluid dynamic Newton theory it was developed the mechanical model that represents two models: one is perfect elastic model and the other one is represented by viscous fluid model.

These viscoleastic models are realized from arcs and dampers systems , the arcs represents the elastic behaviour and the dampers represents the viscous properties of the system. Take into consideration the viscoelastic analogy of Alfred and Lee, where the linear viscoelastic dynamics can be solved with the classical linear elastodynamic equations with Laplace transform applied, there were obtained the displacement equations for viscoelastic bodies. The variational principle of Hamilton can be used for the displacements equations in linear viscoelastic by replacing the Young's elasticity modulus, E, With its Laplace transform $\tilde{E}(s)$.

For mechanical rheological simple models, such as Kelvin-Voight, Standard, Maxwell, Zener or Hooke model connected in parallel way with Maxwell element, $\tilde{E}(s)$ is determined with the following relation:

$$\tilde{E}(s) = \frac{a_0 s + a_1}{b_0 s + b_1}. \tag{1}$$

where a_0, b_0, a_1, b_1 are coefficients and s represents the variable in Laplace transform.

For standard linear solid model which combines the Maxwell model and a Hookean spring in parallel the representation is given in figure 2. A viscous material is modeled as a spring and a dashpot in series with each other, both of which are in parallel with a lone spring. For this model, the governing constitutive relation is:

$$\frac{d\varepsilon}{dt} = \frac{\frac{E_1}{\eta}\left(\frac{\eta}{E_2}\frac{d\sigma}{dt} + \sigma - E_\infty \varepsilon\right)}{E_\infty + E_1} \tag{2}$$

where:

t – time; E_1, E_∞ – the elastic modulus for springs as in figure 2; η – viscosity of the material; σ – the stress in material; ε – the strain in material.

Under a constant stress, the modeled material will instantaneously deform to some strain, and after that it will continue to deform and asymptotically approach a steady state strain.

2. THE INPUT DATA FOR THE NONLINEAR ANALYSIS FOR LINEAR VISCOELASTIC ROD

Considering a cranks and connecting mechanism with elastic elements and one element being a linear viscoelastic type of material, suddenly this element is subjected to a constant axial load applied on one end, the other end being connected to a translational pair.

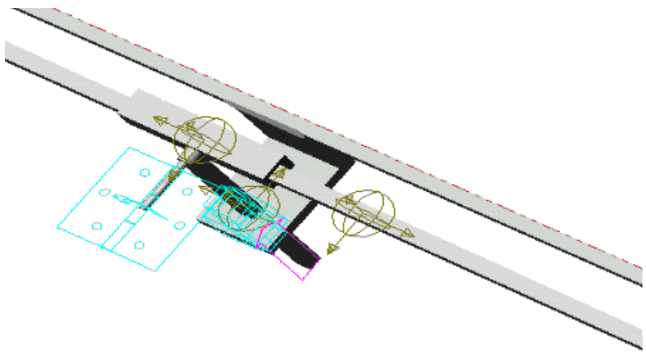

Fig. 1 The elements of the mechanism and the axial load applied to the linear viscoelastic element

The material model is shown in figure 2 where the linear viscoelastic material is represented by a combination of linear springs and a dashpot.

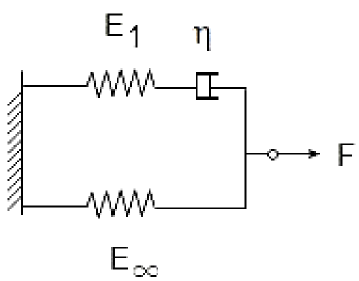

Fig. 2 The linear viscoelastic model for the rod with axial load applied

The extensional relaxation function is given by the equation:

$$E(\tau) = E_{\infty} + E_1 e^{-t/\tau_1^E} \tag{3}$$

where:

E_{∞} - extensional relaxation modulus [MPa] ;
τ_1^E - extensional relaxation time [s].

The time-dependent material behavior inside the code is approximated with a generalized Maxwell model:

$$G(\tau) = G_{\infty} + G_1 e^{-t/\tau_1^E} = G_0\left[1 - g_1\left(1 - e^{-t/\tau_1^G}\right)\right] \tag{4}$$

where:
G - shear relaxation modulus such as:

$$G_0 = 3K_0 E_0 / (9K_0 - E_0) \quad (5)$$

$$G_\infty = 3K_\infty E_\infty / (9K_\infty - E_\infty) \quad (6)$$

$$G_1 = G_0 - G_\infty \quad (7)$$

K - bulk modulus for the material [MPa];

τ_1^G - Shear relaxation time given by the following equations:

$$\left.\frac{dG}{d\tau}\right|_{\tau=0} = \frac{dG}{dE}\left.\frac{dE}{d\tau}\right|_{\tau=0} \quad (8)$$

$$\tau_1^G = \left(\frac{9K_0 - E_0}{9K_0 - E_\infty}\right)\tau_1^E \quad (9)$$

Also, next equations reveal the other parameters:

$$E_0 - E_\infty + E_1 \quad (10)$$

$$\nu_0 = \frac{3K_0 - E_0}{6K_0} \quad (11)$$

$$g_1 = \frac{G_1}{G_0} \quad (12)$$

The input material parameters are:

L=250 mm – the length of the rod; D=25mm - the diameter of the rod.

For all the parameters for the analysis, using the above equations the values a re given in table 1.

Table 1 The values for the parameters of the linear viscoelastic rod

Parameter	Value
E_∞	7 [MPa]
E_1	63 [MPa]
E_0	700 [MPa]
τ_1^E	1 [s]
K	700 [MPa]
G_0	23.6 [MPa]
G_∞	2.336 [MPa]
G_1	21.25 [MPa]
τ_1^G	0.9899 [s]
g_1	0.9
ν_0	0.48

A Heaviside loading function is shown in figure 3. To capture the instantaneous material behavior, a tiny time step (0.005 s) is used in the beginning along with the auto-stepping algorithm during which the rod/bar is relaxed toward its long behavior. Tolerance for creep strain increment CETOL = 5E-4. F_m represents the technological force and has the value 250 N.

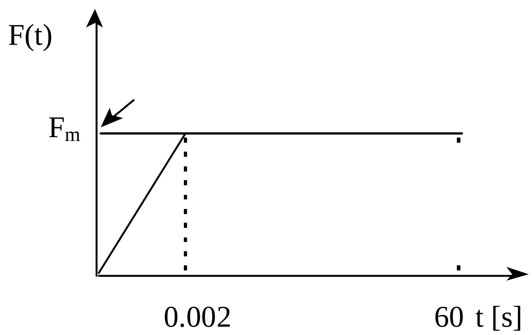

Fig. 3 The Heaviside loading function for the axial load

With all data from above there are realized the geometrical model of the rod, then the material is set giving the constants from the table 1. The real constants are next and the finite element mesh is realized using TRUSS and SOLID elements from the COSMOSM program. The data from figure 3 are necessary for setting the nonlinear analysis parameters.

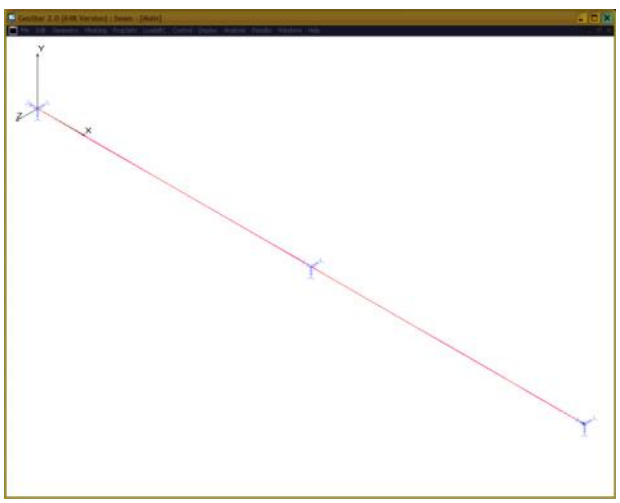

Fig. 4 The finite element model for the rod

3. THE NONLINEAR ANALYSIS RESULTS FOR LINEAR VISCOELASTIC ROD

There were considered that the results for the displacement map and reaction forces map for step 266 at the time t=50 s after two equilibrium iteration performed. The following tables presents the displacement for node 3 at t=50s, the reaction forces for all the three nodes, the axial force, the stress and the total strain for the element, the nonlinear strain as well they are extracted from the file.out of COSMOSM program.

AUTOMATIC SETTING for STEP 266
Starting Time= .498387E+02 Time Inc.(0)= .161277E+00
D i s p l a c e m e n t s f o r s t e p 266 (Time = .5000E+02)
Total number of equilibrium iterations performed = 2

Node	X1	X2	X3	R1	R2	R3

3 .245875+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00

Reaction Forces for step 266 (Time = .5000E+02)

Node	X1	X2	X3	R1	R2	R3
1	-.35625E+02	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
2	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
3	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00

Stress and Strain Printout for Truss Element Group Nu mber 1

Type of Nonlinearity = Material only
Material Model = Viscoelastic (Generalized Maxwell Model)

Element Number	Axial Force	Elastic/ Plastic	Stress	Total Strain	Creep Strain
1	.35625E+02	ELASTIC	.693999E+02	.9938264E-01	.8938267E-01
2	.35625E+02	ELASTIC	.693999E+02	.9938264E-01	.8938267E-01

STNLIST,266,1,0,1,1,2,1

Step 266 Top Face Layer 1

Nonlinear Strains (TOTAL)

Elem	EPSX	EPSY	EPSZ	GMXY	GMXZ	GMYZ
1	9.938e-002	0.000e+000	0.000e+000	0.000e+000	0.000e+000	0.000e+000
2	9.938e-002	0.000e+000	0.000e+000	0.000e+000	0.000e+000	0.000e+000

DISLIST,266,1,1,3,1,0

Node	UX	UY	UZ	RX	RY	RZ
1	0.000e+000	0.000e+000	0.000e+000	0.000e+000	0.000e+000	0.000e+000
2	24.225e-001	0.000e+000	0.000e+000	0.000e+000	0.000e+000	0.000e+000
3	248.75e-001	0.000e+000	0.000e+000	0.000e+000	0.000e+000	0.000e+000

The total strain in time is represented in figure 5 and the X displacement for the node 3 of the rod is represented in figure 6.

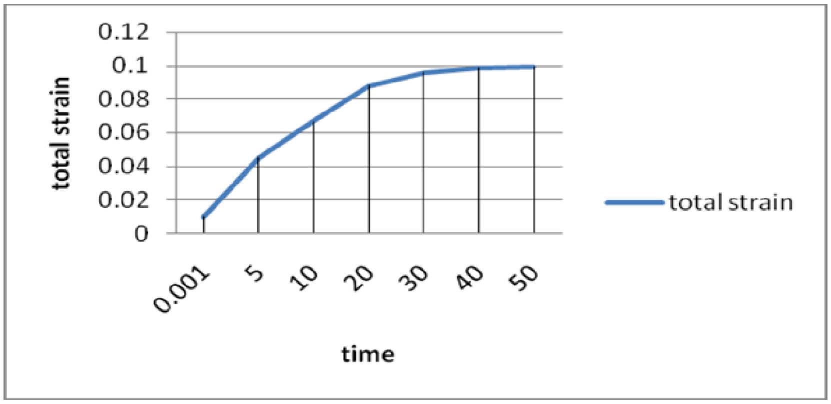

Fig. 5 The total strain in time for the viscoelastic rod

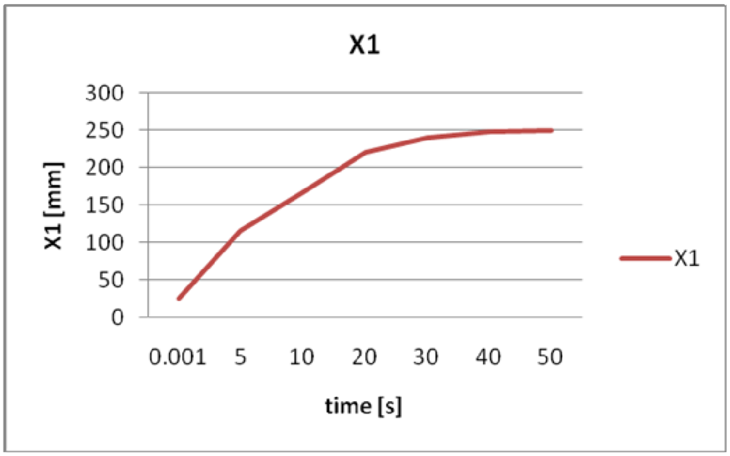

Fig. 6 The displacement in X direction for node 3 in time

All these data were selected from the output file of the nonlinear analysis at the steps 3, 108, 166, 224, 246, 256 and 266.

CONCLUSIONS

The nonlinear analysis of the cinematic elements of the mechanisms is very important for designing these elements. The displacements and deformations of the linear viscoelastic material influence in large percent the high precision work of the mechanism.

The deformations for this material are smaller than the deformations for the same cinematic element in the same work condition realized from steel (assumed elastic material). In this case it is obviously that using the viscoelastic materials (for small speeds of the leading element of the mechanism) goes to improving the best work precision of the cinematic elements.

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